## Subject:-Mathematical Foundations of Computer Science

## Code:- MTCE 603A

## Section A

1 Write all differences between deterministic and non-deterministic finite automata.

2 How a NFA is converted to DFA ? Give example and explain.

3 Construct a DFA equivalent to regular expression ba+(a+bb)a*b

4 Design a FSM with minimum states which accepts all strings over (a,b) such that number of a's is divisible by 2 \& number of b's divisible by 3 .

5 Explain the difference between the transition functions of DFA and NFA.
6 Define Finite Automata

7 Construct a nondeterministic automata accepting the set of all strings over $(a, b)$ ending in $a b$.

8 Construct a finite automata accepting the string having exactly four 1 's over the alphabet $\{0,1\}$

9 Consider the finite state machine whose transition function $\delta$ is given in below table in the form of a transition table. Here, $Q=\left\{q_{0}, q_{1}, q_{2}, q_{3}\right\}, \Sigma=\{0,1\}, F=\left\{q_{0}\right\}$. Give the entire sequence of states for the input string 110001.
Table 1 Transition Function Table

| State | Input |  |
| :---: | :---: | :---: |
|  | 0 | 1 |
|  | $\mathrm{q}_{2}$ | $\mathrm{q}_{1}$ |
| $\mathrm{q}_{1}$ | $\mathrm{q}_{3}$ | $\mathrm{q}_{0}$ |
| $\mathrm{q}_{2}$ | $\mathrm{q}_{0}$ | $\mathrm{q}_{3}$ |
| $\mathrm{q}_{3}$ | $\mathrm{q}_{1}$ | $\mathrm{q}_{2}$ |

10 Construct the state diagram for the finite-state automation $\mathbf{M}=\left(\mathbf{S}, \mathbf{\Sigma}, \boldsymbol{\delta}, \boldsymbol{s}_{\boldsymbol{0}}, \mathbf{F}\right)$, where $\mathbf{S}=\left(\boldsymbol{s}_{\boldsymbol{0}}, \boldsymbol{s}_{\mathbf{1}}, \boldsymbol{s}_{\mathbf{2}}, \boldsymbol{s}_{\mathbf{3}}\right), \mathbf{S}=\{\boldsymbol{a}, \boldsymbol{b}\}, \mathbf{F}=\left\{\boldsymbol{s}_{\boldsymbol{0}}, \boldsymbol{s}_{\mathbf{3}}\right\}$, and the transition function $\boldsymbol{\delta}$ is given the following table.

| State | $a$ | $b$ |
| :---: | :---: | :---: |
| $s_{0}$ | $s_{0}$ | $s_{1}$ |
| $s_{1}$ | $s_{0}$ | $s_{2}$ |
| $s_{2}$ | $s_{0}$ | $s_{0}$ |
| $s_{3}$ | $s_{2}$ | $s_{1}$ |

11 The transition table of a Non-deterministic finite automaton $M$ is defined by below table. Construct a deterministic finite automaton equivalent to M .
Table 2: Transition table

| State | Input |  | 2 |
| :---: | :---: | :---: | :---: |
|  | 0 | 1 |  |
| $\rightarrow \mathrm{q}_{0}$ | $\mathrm{q}_{1} \mathrm{q}_{4}$ | $\mathrm{q}_{4}$ | $\mathrm{q}_{2} \mathrm{q}_{3}$ |
| $\mathrm{q}_{1}$ |  | $\mathrm{q}_{4}$ |  |
| $\mathrm{q}_{2}$ |  |  | $\mathrm{q}_{2} \mathrm{q}_{3}$ |
| (9) |  | $\mathrm{q}_{4}$ |  |
| $\mathrm{q}_{4}$ |  |  |  |

12 What do you mean by Finite automation with output. Explain the way in which Moore and Mealy machine differ from each other. Also prove that if $\mathrm{M} 1=\left(\mathrm{Q}, \sum, \Delta, \delta, \lambda, \mathrm{q}\right)$ is a moore machine then there is a mealy machine M2 equivalent to M1.

## Section B

1 Show that the regular expression $(a+b) * a(a+b) * b(a+b)^{*}$ is equivalent to $(a+b) * a b(a+b) *$ in the sense that they define the same language.

2 Find a cfg that generate the following language over alphabet $\sum=(a, b)$
i) all string that end in $b$ and have an even number of $b$ 's in total
ii) all string of odd length

3 Find the language generated by the grammar
$S->A B, A->A 1 / 0, B->2 B / 3$
4. Convert the given grammar into GNF
$S->A B, A->B S / b, B->S A / a$

5 Construct a PDA named A equivalent to the following context free grammar $\mathrm{S}->0 \mathrm{BB}, \mathrm{B}->0 \mathrm{~S} / 1 \mathrm{~S} / 0$ Test whether $010^{4}$ is in $N(A)$.

6 Explain the application of pumping lemma. Give suitable example.
7 What is minimization algorithm.

## Section C

1 Construct a PDA accepting the set of all even length palindromes over the $\{a, b\}$ by empty store

2 Construct a turing machine that can accept set of all even palindromes over $\{0,1\}$
3 Design a T.M to recognize the language $\left\{a^{n} b^{n} c^{m} \mid n, m>=1\right\}$

4 Convert the following grammar in to Greibach Normal Form (GNF)
$\mathrm{S} \rightarrow \mathrm{aSa} / \mathrm{bSb} / \mathrm{a} / \mathrm{b} / \mathrm{aa} / \mathrm{bb}$.
5. Find a reduced grammer equivalent to the grammar G whose Productions are:
$S \rightarrow A B$
$A \rightarrow a$
$B \rightarrow b$
$E \rightarrow C$
6. Explain CNF and its Lemmas in detail. Find a Grammar in CNF equivalent to the grammar
$S \quad \sim S \mid[S \quad$ ) S]|p|q$\quad$ (S being the only variable)
7. Describe the context free and context sensitive grammar. Also explain chemosky normal form.
8. Describe the system for the pushdown automation. Also write purpose of pushdown automation.
9. Construct pushdown Automata for the accept all the strings over alphabet $\{a, b\}$ with exactly twice as many a's as b's.
10. Construct pushdown Automata for the To accept the set of palindromes over $\{a, b\}$.

## Section D

1 Show that following functions are primitive recursive
I) $q(x, y)=$ the quotient obtained when $x$ is divided by $y$.
II) $r(x, y)=$ remainder obtained when $x$ is divided by $y$

2 Write Short notes on Chomsky hierarchy of a grammar
3 Write Short notes on Halting problem of Turing Machine.
4 Write Short notes on PCP problem
5 Determine whether all the strings in each of these sets are recognized by the Deterministic finite-state automation given below.

(i) $\{a\}^{*}$
(ii) $\{a\}\{a\}^{*}$
(iii) $\left\{b \mid\{a\}^{*}\right.$
(iv) $\{a b\}^{*}$
(v) $\{a\}^{*}\{b\}^{*}$
(vi) $\{b\}\{a, b\}^{*}$
6. Explain the basic model of Turing machine. Design a TM to accept language $\mathrm{L}=\left\{\mathrm{WCW} \mid \mathrm{W}\right.$ in $\left.(\mathrm{a}+\mathrm{b})^{+}\right\}$.

7 Explain the Regular Grammar
8 Explain the Unrestricted Grammar.
9 Write the procedure to convert an NFA with $\varepsilon$ moves to an equivalent NFA with at $\varepsilon$ moves and apply it to the following NFA

10 Design a turing machine that takes any unary input of even number of $1 s$ and halts when first half is converted in to number of 0 s

